3.1 Rechnerisches Lösen von Polynomgleichungen

Quadratische Gleichungen

1a) $2x^2 + 3x - 2 = 0$

$$x_{1/2} = \frac{-3 \pm \sqrt{3^2 - 4 \cdot 2 \cdot (-2)}}{4}$$

$$= \frac{-3 \pm 5}{4}$$

$x_1 = \frac{1}{2}$

$x_2 = -2$

b) $-x^2 - 2x + 3 = 0$

$$x_{1/2} = \frac{2 \pm \sqrt{(-2)^2 - 4 \cdot (-1) \cdot 3}}{-2}$$

$$= \frac{2 \pm 4}{-2}$$

$x_1 = -3$

$x_2 = 1$

2a) $x^2 - 12x - 36 = 0$

$$x_{1/2} = \frac{12 \pm \sqrt{(-12)^2 - 4 \cdot 1 \cdot (-36)}}{2}$$

$x_1 = 6$

2b) $x^2 - 10x + 25 = 0$

$$x_{1/2} = \frac{10 \pm \sqrt{10^2 - 4 \cdot 1 \cdot 25}}{2}$$

$x_1 = 5$

3a) $9x^2 - 6x + 2 = 0$

$$x_{1/2} = \frac{6 \pm \sqrt{(-6)^2 - 4 \cdot 9 \cdot (-2)}}{18}$$

3b) $x^2 - 2x + 3 = 0$

$$x_{1/2} = \frac{2 \pm \sqrt{(-2)^2 - 4 \cdot 1 \cdot 3}}{2}$$

4a) $2x^2 - 24 = 0$

$2x^2 = 24$

$x^2 = 12$

$x_{1/2} = \pm\sqrt{12}$

4b) $0{,}5x^2 - 4{,}5 = 0$

$0{,}5x^2 = 4{,}5$

$x^2 = 3$

$x_{1/2} = \pm\sqrt{3}$

5a) $3 \cdot (x - 0{,}5)(0{,}75 - x) = 0$

$x_1 = 0{,}5$

$x_2 = -0{,}75$

5b) $1{,}5\,(2x - 4)(3 - 0{,}5x) = 0$

$x_1 = 4$

$x_2 = -3$

Ausklammern

a) $\frac{1}{3}x^3 - \frac{7}{3}x^2 - \frac{8}{3}x = 0$

$x\left(\frac{1}{3}x^2 - \frac{7}{3}x - \frac{8}{3}\right) = 0$

$x_1 = 0 \quad \vee \quad x^2 - 7x - 8 = 0$

$x_{1/2} = \frac{7 \pm \sqrt{(-7)^2 - 4 \cdot 1 \cdot (-8)}}{2}$

$x_{1/2} = \frac{7 \pm 9}{2}$

$x_1 = 8$

$x_2 = 1$

b) $3x^4 - 9x^3 = 0$

$x^3(3x - 9) = 0$

$x^3 = 0 \quad \vee \quad 3x - 9 = 0$

$x_1 = 0 \qquad 3x = 9$

$x_2 = 3$

c) $x - x^2 = 2x^3$

$0 = 2x^3 + x^2 - x$

$0 = x(2x^2 + x - 1)$

$x_1 = 0 \quad \vee \quad 2x^2 + x - 1 = 0$

$x_{1/2} = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 2 \cdot (-1)}}{4}$

$x_{1/2} = \frac{-1 \pm 3}{4} \qquad x_1 = 1/2$

$x_2 = -1$

Direktes Wurzelziehen

a) $-x^3 + 64 = 0$

$64 = x^3$

$\sqrt[3]{64} = x$

$4 = x$

b) $\frac{1}{2}x^5 - \frac{1}{2} = 0$

$x^5 - 1 = 0$

$x^5 = 1$

$x = 1$

c) $2x^6 + 22 = 2$

$2x^6 = -20$ ↯

Substitution

a) $x^4 - 8x^2 + 7 = 0$ $\qquad z = x^2$

$z^2 - 8z + 7 =$

$$z_{1/2} = \frac{8 \pm \sqrt{(-8)^2 - 4 \cdot 1 \cdot 7}}{2}$$

$$z_{1/2} = \frac{8 \pm 6}{2} \qquad z_1 = 7 \quad z_2 = 1$$

$7 = x^2$ $\qquad 1 = x^2$

$\pm\sqrt{7} = x_{1/2}$ $\qquad 1 = x_3$

b) $x^8 + 2x^4 + 1 = 0$ $\qquad z = x^4$

$z^2 + 2z + 1 = 0$

$$z_{1/2} = \frac{-2 \pm \sqrt{2^2 - 4 \cdot 1 \cdot 1}}{2}$$

$z_1 = -1$

$-1 = x^4$ ↳

c) $\frac{1}{2}x^6 - \frac{7}{2}x^3 - 4 = 0$

$x^6 - 7x^3 - 8 = 0$ $\qquad z = x^3$

$z^2 - 7z - 8 = 0$

$$z_{1/2} = \frac{7 \pm \sqrt{(-7)^2 - 4 \cdot 1 \cdot (-8)}}{2}$$

$$z_{1/2} = \frac{7 \pm 9}{2} \qquad z_1 = 8 \quad z_2 = -1$$

$8 = x^3$

$\sqrt[3]{8} = x_1$

$2 = x_1$

$-1 = x^3$

$-1 = x_2$